Permutation & Combination

1. Factorial Notation:

Let *n* be a positive integer. Then, factorial *n*, denoted *n*! is defined as:

**n! = n(n - 1)(n - 2) ... 3.2.1.**

**Examples:**

We define **0! = 1**.

4! = (4 x 3 x 2 x 1) = 24.

1. Combination:

Combination represent nCr, can be defined as the number of ways in which r things at a time can be SELECTED from amongst n things available for selection.

nCr = Number of combinations (selections) of n things taken r things at a time.

nCr = n!/ [r!(n-r)!]; where n r (n is greater than or equal to r)

Some typical situations:

1. Selection of people of a team, a party, a job an office etc. (e.g. selection of a football team of 11 from 16 member = 16C11
2. Selection of a set of objects (like letters, hats, points, pants, shirts etc.) from amongst another set available.
3. Permutation

Permutation (represented by nPr) can be defined as the number of ways in which r things at a time can be SELECTED & ARRANGED at a time from amongst r things.

The key word here is ARRANGEMENT. Hence understand here that the order in which the r things are arranged has critical importance in the counting of permutations.

nPr = Number of permutations (arrangements) of n things taken r at a time.

nPr  = n!/(n-r)!; where n r

Some typical situations:

1. Putting distinct objects/people in distinct places, e.g. making people sit, putting letters in envelopes, finishing order in horse race etc.
2. Selection of batting order of a cricket team of 11 from 16 members = 16P11
3. Number of ways in which n different beads can be arranged in a necklace is ½ X (n-1)!
4. Understanding SELECTION & ARRANGEMENTS

SELECTION:

Suppose we have 3 men A, B, C out of which 2 men have to be selected for 2 posts. This can be done in the following ways: AB, AC or BC. Physically they can be counted as 3 distinct selections. This value can also be got be using 3C2

ARRANGEMENTS:

Suppose we have 3 men A, B, C out of which 2 men have to be arranged , this can be done in 6 ways: AB, BA, AC, CA, BC, CB. This value can also be get using 3P2

Relationship between Permutation & Combination:

nPr  = r! X nCr (Or in other words: The permutation or arrangement of r things out of n is nothing but the selection of r thing out of n followed by the arrangement of r selected things amongst themselves.

**Important things for questions solving:**

1. Number of permutation (arrangements) of n different things taken all at a time = n!
2. In case of circular arrangement clockwise and anticlockwise arrangements are distinct arrangements, therefore total arrangement = (n-1)! (If clockwise & anticlockwise observation is same then no. of arrangements = ½ X (n-1)!)
3. If out of n arrangements P1, P2 & P3 are alike then number of arrangements

=

1. Number of selection from n identical things = 1
2. Total number of selection of zero or more things out of n different things =

nC0 +nC1+nC2+…………..+ nCn =

If at least 1 has to be selected nC1+nC2+…………..+ nCn = – 1

1. If there are three things to do and there are M ways of doing first thing, N ways of doing 2nd thing & P ways of doing 3rd thing, then there will be M X N X P ways of doing all three things together. The works are mutually inclusive.
2. “Distributing” n things to r people

= n+r-1Cr-1 (When each person may get any number of things)

= n-1Cr-1 (When each person may get any number of things excluding “0”)

1. If “n Different” things has be distributed among “r different person” then number of ways of doing it =
2. Number of ways of dividing m+n different things in 2 groups containing m and n things respectively = (m+n)!/m!n!
3. Number of ways of dividing 2n different things in 2 groups containing n things = 2n!/n!n!2!
4. Number of selection of r things out of n different things, when k are
5. Always included = n-kCr-k
6. Always Excluded = n-kCr
7. Number of ways of selecting 0 to n things out of n things where p are of 1 type, q are of other type and r of another type = (p+1)(q+1)(
8. Number of terms in series = n+m-1Cn-1
9. Number of ways of shake hands by n people = nC2 (Could also be used where 2 teams, and anywhere where 2 groups/people required to perform any action.)
10. If a square is made by n horizontal and n vertical lines then total number of square

=

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Probability

1. **Experiment:**

An operation which can produce some well-defined outcomes is called an experiment.

1. **Random Experiment:**

An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

**Examples:**

* 1. Rolling an unbiased dice.
  2. Tossing a fair coin.
  3. Drawing a card from a pack of well-shuffled cards.
  4. Picking up a ball of certain color from a bag containing balls of different colors.

**Details:**

* 1. When we throw a coin, then either a Head (H) or a Tail (T) appears.
  2. A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.
  3. A pack of cards has 52 cards.

It has 13 cards of each suit, name **Spades, Clubs, Hearts and Diamonds**.

Cards of spades and clubs are **black cards**.

Cards of hearts and diamonds are **red cards**.

There are 4 honours of each unit.

There are **Kings, Queens and Jacks**. These are all called **face cards**.

1. **Sample Space:**

When we perform an experiment, then the set S of all possible outcomes is called the **sample space**.

**Examples:**

* 1. In tossing a coin, S = {H, T}
  2. If two coins are tossed, the S = {HH, HT, TH, TT}.
  3. In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.

1. **Event:**

Any subset of a sample space is called an **event**.

1. **Probability of Occurrence of an Event:**

Let S be the sample and let E be an event.

Then, E http://www.indiabix.com/_files/images/aptitude/1-sym-eps.gif S.

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| http://www.indiabix.com/_files/images/aptitude/1-sym-tfr.gif P(E) = | *n*(E) | . |
| *n*(S) |

1. **Results on Probability:**
   1. P(S) = 1
   2. 0 http://www.indiabix.com/_files/images/aptitude/1-sym-leq.gif P (E) http://www.indiabix.com/_files/images/aptitude/1-sym-leq.gif 1
   3. P(http://www.indiabix.com/_files/images/aptitude/1-sym-phi.gif) = 0
   4. For any events A and B we have : P(A http://www.indiabix.com/_files/images/aptitude/1-sym-uni.gif B) = P(A) + P(B) - P(A http://www.indiabix.com/_files/images/aptitude/1-sym-vec.gif B)
   5. If A denotes (not-A), then P(A) = 1 - P(A).

**Logarithm:**

If *a* is a positive real number, other than 1 and *am* = *x*, then we write:  
***m* = loga*x*** and we say that the value of log *x* to the base *a* is *m*.

**Examples:**

(i). 103 1000   http://www.indiabix.com/_files/images/aptitude/1-sym-imp.gif   log10 1000 = 3.

(ii). 34 = 81   http://www.indiabix.com/_files/images/aptitude/1-sym-imp.gif   log3 81 = 4.

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| (iii). 2-3 = | 1 | http://www.indiabix.com/_files/images/aptitude/1-sym-imp.gif   log2 | 1 | = -3. |
| 8 | 8 |

(iv). (.1)2 = .01   http://www.indiabix.com/_files/images/aptitude/1-sym-imp.gif   log(.1) .01 = 2.

1. **Properties of Logarithms:**

1. loga (*xy*) = loga *x* + loga *y*

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| 2. loga | http://www.indiabix.com/_files/images/aptitude/1-sym-oparen-h1.gif | *x* | http://www.indiabix.com/_files/images/aptitude/1-sym-cparen-h1.gif | = loga *x* - loga *y* |
| *Y* |

3. logx *x* = 1

4. loga 1 = 0

5. loga (*xn*) = *n*(loga *x*)

|  |  |
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| 6. loga *x* = | 1 |
| logx *a* |

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| 7. loga *x* = | logb *x* | = | log *x* | . |
| logb *a* | log *a* |

1. **Base Change Rule**
   * 1. log c to the base is equal to
2. **The Characteristic and Mantissa of a Logarithm**

The logarithm of number consists of two parts: the integral part and the decimal part. The integral part is known as the characteristic and the decimal part is called mantissa

For example,

In log 3257 = 3.5128, the integral part is 3 and mantissa is .5128

Note:- Mantissa is always to be written as positive. So in case the value of logarithm o f a number is negative, subtract 1 from the integral part and add 1 to the decimal part.

Thus, -3.4328 = -3 – 0.4328

= (-3 – 1) + (1 – 0.4328) = -4 + .5672

So the mantissa is .5672

The characteristic may be positive or negative. When characteristic is negative it is represented by putting a bar on the number.

Thus instead of -4, we write

Hence we may write -4 + .5672 =

**Set Theory**

A set is well-defined collection of objects. The objects, which belong to the set, are called elements of the set. A set is denoted by a capital letter, and the elements of a set are denoted by small letter.

Greek letter” is used to donate than an element ‘belong to a set’.

**Type of Set**

1. Finite Set: A set is finite set if it consist of definite number of elements.

Example: V is set of vowels in English language V = {a,e,i,o,u}

1. Infinite Set: A set in which elements are impossible to count

Example: N {n│ n is a natural number}

1. Empty Set: The set, which contains no elements at all, is called an empty or null set. It is represented as {} or
2. Subsets: A set ‘A’ is a subset of set ‘B’ if and only if every element of A is an element of B.
3. Universal Set: Consider the sets A, B, C and D. Any set of which A, B, C and D are subsets is called universal set. The Universal set is denoted by U.

Example: The universal set is the set of real numbers, while the set of natural numbers, whole numbers, Integers and Rational numbers.

**Operations of Sets**

1. Union of Sets: The union of sets A and B is a set which consists of all the elements of A and B.

A U B {x │ x A or x B}

Some Important Points

* + 1. A U B = B U A
    2. A U = A

1. Intersections of Sets

The intersections of two sets A and B is a set which consists of all the common elements of A and B. The symbol denotes intersection.

A B {x │ x A and x B}

1. Complement of a Set:

Complement of set A with respect to the universal set U is the set of all those elements of U which are not the elements of A. It is denoted by A’ or

A’ = {x │ x U and x ∉ B}

Some Important Points

* + 1. Complement of the universal set is the null set and vice versa.
    2. (A’)’ = A
    3. A U A’ = U (i.e. Universal Set)

1. **Difference of Sets**

If A and B are 2 sets, then the set of all elements which belong to A but do not belong to B is called the difference of sets A and B, and is denoted by A - B

A – B = {x │ x A and x ∉ B}

Some Important Points

* + 1. A – B ≠ B – A

**Important Formulas:**

If  *= intersection* and *U = union*.

* + - 1. P(A U B U C) = P(A) + P(B) + P(C) – P(A B) – P(A C) – P(B C) + P(A B C)
      2. To find the number of people of people in exactly 1 set:

P(A) + P(B) + P(C) – 2P(A B) – 2P(A C) – 2P(B C) + 3P(A B C)

* + - 1. To find the number of people in *exactly* two sets:

P(A B) + P(A C) + P(B C) – 3P(A B C)

* + - 1. To find the number of people in exactly 3 sets:

P(A B C)

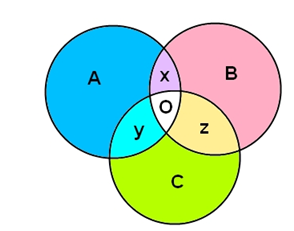
* + - 1. To find the number of people in 2 or more sets:

P(A B) + P (A C) + P(B C) – 2P (A B C)

* + - 1. To find the number of people in at least 1 set:

P(A) + P(B) + P(C) – P(A B) – P(A C) – P(B C) + 2P(A B C)

* + - 1. To find the number of people in *at least* one set:

[](http://grockit.com/blog/wp-content/uploads/2011/01/formula-Set.png)

For questions involving set theory, it may be helpful to make a Venn diagram to visualize the solution.

To find the union of all set:                 (A + B + C + X + Y + Z + O)

Number of people in *exactly* one set:     (A + B + C)

Number of people in *exactly* two of the sets: (X + Y + Z)  
Number of people in *exactly* three of the sets: O

Number of people in *two or more* sets: (X + Y + Z  + O)

**Functions**

 In mathematics, a function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output. An example is the function that relates each real number x to its square x². [Wikipedia](http://en.wikipedia.org/wiki/Function_(mathematics))

  Related topics

**Derivative** in calculus, a quantity indicating how a function changes when the values of its inputs change. [Wikipedia](http://en.wikipedia.org/wiki/Derivative_(disambiguation))

**Explore**: [Derivative](https://www.google.co.in/search?espv=210&es_sm=122&biw=1517&bih=714&q=Derivative&stick=H4sIAAAAAAAAAGOovnz8BQMDAw8HsxKHfq6-gZFZtkG3p9ahQiutV1d5tioddd7VdkSloB4AS123wSkAAAA&sa=X&ei=3BA_U9jJGoWMrQeclIGwAg&ved=0CJwBEPcxMBM)

While **polynomial** functions are defined for all values of the variables, a rational function is defined only for the values of the variables for which the denominator is not null. [Wikipedia](http://en.wikipedia.org/wiki/Polynomial)

**Explore**: [Polynomial](https://www.google.co.in/search?espv=210&es_sm=122&biw=1517&bih=714&q=Polynomial&stick=H4sIAAAAAAAAAGOovnz8BQMDAw8HsxKHfq6-gWlhRfblurPF15dnWetLNZpv9RVeN9NLRBwA0gcr1ykAAAA&sa=X&ei=3BA_U9jJGoWMrQeclIGwAg&ved=0CKABEPcxMBM)

A function is **continuous** at a point iff for any neighborhood of its image the preimage is again a neighborhood of that point: [Wikipedia](http://en.wikipedia.org/wiki/Continuous_function)

**Explore**: [Continuous function](https://www.google.co.in/search?espv=210&es_sm=122&biw=1517&bih=714&q=Continuous+function&stick=H4sIAAAAAAAAAGOovnz8BQMDAw8HsxKHfq6-gWFJVbzypApJ1ST16P-mbuun3WT9pr1wfxIAICkZLykAAAA&sa=X&ei=3BA_U9jJGoWMrQeclIGwAg&ved=0CKQBEPcxMBM)